

Proposals for dissertation projects

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Project 1: Turbulence in plasmas

I. CONTEXT

Moving plasmas are ubiquitous in astrophysics, in stellar clouds, neutron stars, or as a model for ejecta like solar wind for example. Their evolution is described by the so-called MagnetoHydroDynamics (MHD) equations: As the charged particles are put into motion, they generate a magnetic field that, in turn, will also influence the motion of all the charged particles in the plasma. These phenomena are called the dynamo effect and the magnetic reconstruction. Thus, understanding how turbulence develops in these systems is crucial to understand how energy flows between the different scales, from the macroscopic ones to the microscopic ones, and reversely, as well as how the total energy is split between the fluid's kinetic energy and magnetic energy.

If one considers a plasma in a tokamak, similar problems are also at play: The charged particles are put into a global magnetic field, but the dynamo effect and magnetic reconstruction are very much still taking place. In such a problem, understanding how energy flows is no less important.

Unfortunately, the fundamental equations of MHD are very complex, so that most of the works that have been done today are restricted to numerical simulations, which can be a source of problem since very high resolutions are often required, and artifacts can be present in the results, obscuring the real sources of the observed effects. This project aims at building an analytical model of turbulence in a plasma, or, more precisely, make the first steps toward the realisation of such a model.

II. PROJECT

Recent studies conducted in the group of L. Canet (Grenoble, France) have shown how advanced renormalisation group tools can be used to describe with high precision the energy cascade in Newtonian fluid turbulence. In particular, they have shown that their formalism adapts naturally to the Kraichnan model of passive scalar advection in the turbulent fluid (the passive scalar can be a temperature, the concentration of some species, or salinity for salted water for example). The project aims at decomposing the MHD problem into smaller subproblems that are to be solved step by step:

- 1) Describe the advection of a passive *vector* field by the turbulent flow.
- 2) Change the form of the advection term to use the MHD one instead. In this model, the magnetic field evolution equation is respected, but the backreaction term is not present in the Navier-Stokes flow.

Of course, if such projects are completed before the end of the dissertation period, the extension to the full MHD equations will be discussed, but this point is rather advanced and is therefore not considered as an objective of the project.

III. REQUIREMENTS

A strong background in field theory (both quantum and statistical) is required. Advanced knowledge about renormalisation will be provided at the start of the working period. Experience with turbulence energy cascade scalings and background knowledge about turbulence in Newtonian fluids is a plus.

Project 2: Turbulence in cosmological fluids

IV. CONTEXT

The study of the matter-gravity interaction in an expanding universe can be described as a relativistic fluid. The presence of local fluctuations, or heterogeneities in such a fluid can be accounted for by a stochastic contribution to the energy-momentum tensor. In that case, the flow equations of the fluid present turbulent patterns. This project aims at studying how the energy flows, in such a fluid, between the different scales, and how the structures of different sizes can form in an expanding universe by the development of a field-theory model of the relativistic turbulent fluid.

V. PROJECT

Recent studies conducted in the group of L. Canet (Grenoble, France) have shown how advanced renormalisation group tools can be used to describe with high precision the energy cascade in Newtonian fluid turbulence. In particular, their studies have shown that symmetries strongly constraint the evolution of the observables in such a theory, at a fully non-perturbative level, and quite independently of the model used for the study of the renormalisation group flow. A more recent investigation of turbulence in non-Newtonian fluids by O. Coquand (Perpignan, France) have shown that a very subtle change of the symmetry structure is sufficient to generate significant changes, such as a change of universality class for the evolution of the velocities correlation functions.

This project will lead the student to investigate how the previous work can be applied to a universe expanding according to the Friedmann-Lemaître-Robertson-Walker (FLRW) model. In particular, the change from a Galilean to a Lorentzian type of symmetry in space-time has a dramatic effect on the non-perturbative structure of the renormalised vertex hierarchy.

If the project goes on well, investigation of a more general model of expanding universe could be investigated, but this is not fixed as an objective for the project.

VI. REQUIREMENTS

A strong background in field theory (both quantum and statistical) is required. Basic knowledge in cosmology, on the Friedmann-Lemaître-Robertson-Walker (FLRW) model is required. Advanced knowledge about renormalisation will be provided at the start of the working period. Experience with turbulence energy cascade scalings and background knowledge about turbulence in Newtonian fluids is a plus. Knowledge of more advanced notions in cosmology is a plus.